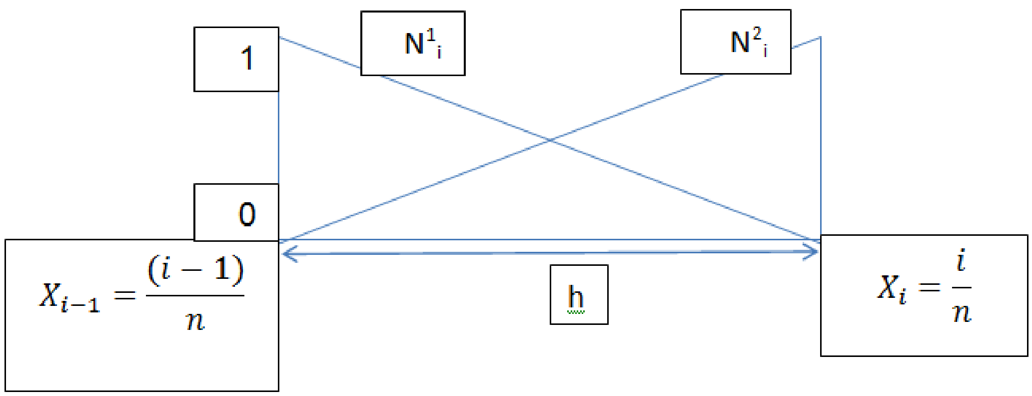
PEng-Consulting

Simple Case - 1D Rod

Rev.00



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1. **Analytical Solution EXACT sol:**

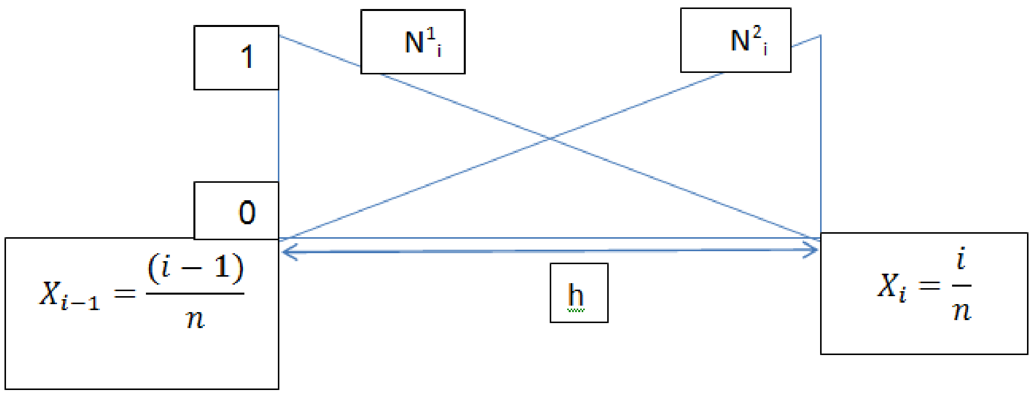
$$Eq.\ \frac{\partial^2 u}{\partial x^2} + x = 0$$

B.C. $u(1) = 0$

$$\frac{\partial u}{\partial x} = 0$$

$$u(x) = \int_x^1 \left\{ \int_0^y \{x\} dz \right\} dy = \frac{1}{6}(1 - x^3)$$

2 **FE method:**



$\{F\} = [\text{K}] \cdot \{d\}$

- F= Force vector
- K = Stiffness Matrix
- d= displacement vector
- n=element numbers
- i=number of global nodes

Shape function:

$$N_i^1 = -n_{nel}(x - \frac{i-1}{n}) + 1$$

$$N_i^2 = n_{nel}(x - \frac{i-1}{n})$$

Forces on Element:

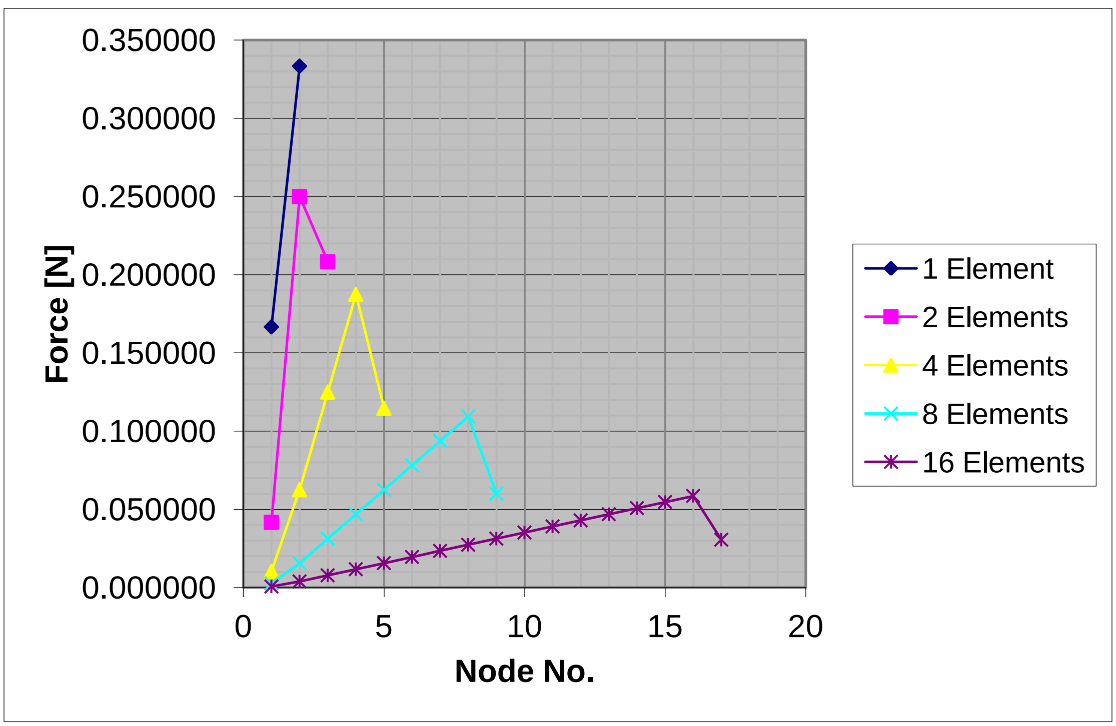
$$F_{i-1} = \int_{(i-1)/n}^{i/n} N_i^1 x dx = -\frac{n_{nel} x^3}{3} + \frac{i x^2}{2} \Big|_{(i-1)/n}^{i/n}$$

$$F_i = \int_{(i-1)/n}^{i/n} N_i^2 x dx = \frac{n_{nel} x^3}{3} - \frac{(i-1) \cdot x^2}{2} \Big|_{(i-1)/n}^{i/n}$$

$$F^e = \frac{h^e}{6} \begin{bmatrix} 2 f(x_i) + f(x_{i-1}) \\ f(x_i) + 2 f(x_{i-1}) \end{bmatrix}$$

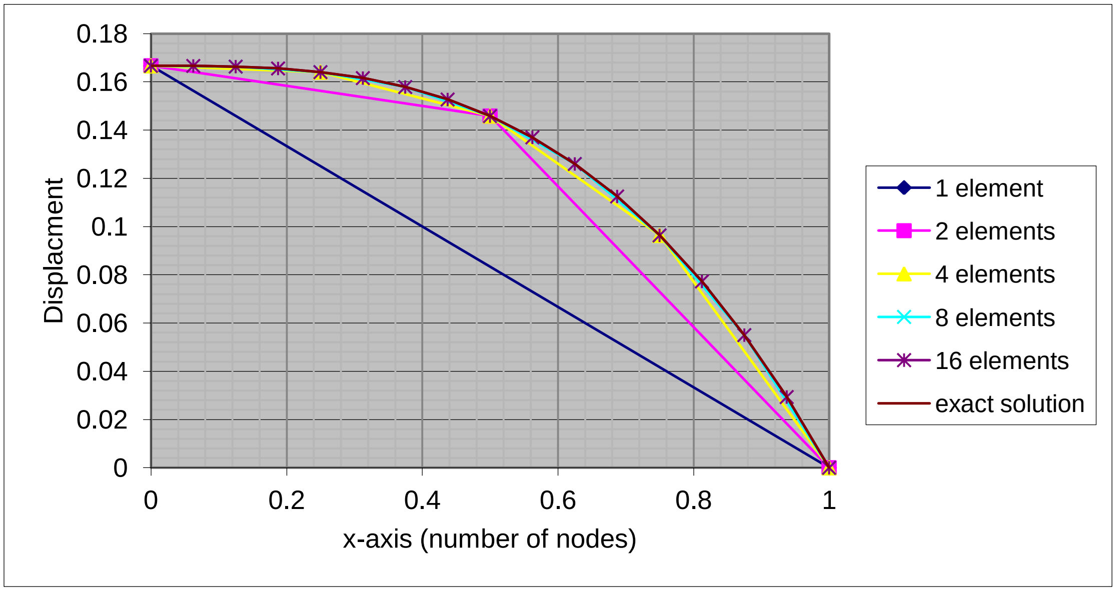
Summarized Forces by ANSYS APDL

no. of elm \ node no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.166667	0.333330															
2	0.041667	0.250000	0.208333														
4	0.010417	0.062500	0.125000	0.187500	0.114583												
8	0.002604	0.015625	0.031250	0.046875	0.062500	0.078125	0.093750	0.109370	0.059901								
16	0.000651	0.003906	0.007813	0.011719	0.015625	0.019531	0.023438	0.027344	0.031250	0.035156	0.039063	0.042969	0.046875	0.050781	0.054688	0.058594	0.030599



Summarized Displacement by ANSYS APDL:

no. of elm \ node no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.166667	0															
2	0.166667	0.145833	0														
4	0.166667	0.164063	0.145833	0.096354	0												
8	0.166667	0.166341	0.164063	0.157878	0.145833	0.125977	0.096354	0.055013	0								
16	0.166667	0.166626	0.166341	0.165568	0.164063	0.161581	0.157878	0.15271	0.145833	0.137004	0.125977	0.112508	0.096354	0.077271	0.055013	0.029338	0



MATLAB

Nel=1,2,4,8,16

```
clc;
    clear all;
    Nel=input('Enter no. of elements: ');
    Fnode = zeros(Nel+1,1);

    for i=1:Nel
      if i==1
      x1=(i-1)/Nel;
      x2=(i)/Nel;
      Fnode(i)= -Nel*x2^3/3+(i*x2^2)/2-(-Nel*x1^3/3+(i*x1^2)/2);

    end
      if i==Nel
       x1=(i-1)/Nel;
       x2=(i)/Nel;
    Fnode(i)=Nel*x2^3/3+(x2^2-i*x2^2)/2-(Nel*x1^3/3+(x1^2-i*x1^2)/2);

      end
        if (i>1 && i<(Nel+1))
      x0=(i-1)/Nel;
      x1=(i)/Nel;
      x2=(i+1)/Nel;
 sum1= Nel*x1^3/3+(x1^2-(i-1)*x1^2)/2-(Nel*x0^3/3+(x0^2-(i-1)*x0^2)/2);
     sum2=-Nel*x2^3/3+(i*x2^2)/2-(-Nel*x1^3/3+(i*x1^2)/2);
     Fnode(i)=sum1+sum2;

        end
     end
```

3 **Plot the convergence of the error in the energy norm as a function of the mesh parameter on a log-log scale.**